

CHAPTER 11

ANTENNAS

" CHAPTER- 11 ——— ANTENNAS " "

Exercise 11.1 $\nabla \cdot \vec{D} = \rho$ $\vec{D} = \epsilon \vec{E} \Rightarrow \nabla \cdot \vec{E} = \rho / \epsilon$

But $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$. Thus $\nabla \cdot (\nabla V + \frac{\partial \vec{A}}{\partial t}) = -\frac{\rho}{\epsilon}$

Hence, $\nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \vec{A}) = -\frac{\rho}{\epsilon}$

Since $\nabla \cdot \vec{A} = -\mu \epsilon \frac{\partial V}{\partial t}$, we have $\nabla^2 V - \mu \epsilon \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon}$

Exercise 11.2

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2} = -\mu J_x$$

$$\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2} = -\mu J_y$$

$$\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2} = -\mu J_z$$

Exercise 11.3

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \frac{\partial}{\partial r} \left(\frac{\tilde{G}}{r} \right) \right] + \beta^2 \frac{\tilde{G}}{r} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[r^2 \left(\frac{1}{r} \frac{\partial \tilde{G}}{\partial r} - \frac{1}{r^2} \tilde{G} \right) \right] + \beta^2 \frac{\tilde{G}}{r} = 0$$

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left[r \frac{\partial \tilde{G}}{\partial r} - \tilde{G} \right] + \beta^2 \frac{\tilde{G}}{r} = 0$$

$$\frac{1}{r^2} \left[r \frac{\partial^2 \tilde{G}}{\partial r^2} + \frac{\partial \tilde{G}}{\partial r} - \frac{\partial \tilde{G}}{\partial r} \right] + \beta^2 \frac{\tilde{G}}{r} = 0$$

Thus, if $r \neq 0$, $\frac{\partial^2 \tilde{G}}{\partial r^2} + \beta^2 \tilde{G} = 0$

Exercise 11.4 From (11.35) $\frac{1}{\beta r} = 1$

Since $\beta = \frac{2\pi}{\lambda}$, $r = \frac{1}{\beta} = \frac{\lambda}{2\pi} \approx \frac{\lambda}{6}$

Exercise 11.5

$$\tilde{A} = \frac{\mu \tilde{I} l}{4\pi r} e^{-j\beta r} [\cos\theta \vec{a}_r - \sin\theta \vec{a}_\theta]$$

$$\vec{B} = \nabla \times \tilde{A} \Rightarrow \tilde{H} = \frac{1}{\mu} (\nabla \times \tilde{A}) = j \frac{\beta \tilde{I} l}{4\pi r} \left(1 + \frac{1}{j\beta r}\right) \sin\theta e^{-j\beta r} \vec{a}_\phi$$

$$\begin{aligned} \tilde{E} = \frac{1}{j\omega\epsilon} (\nabla \times \tilde{H}) &= \frac{\beta \tilde{I} l}{4\pi\omega\epsilon r^2} \left(1 + \frac{1}{j\beta r}\right) (2\cos\theta) e^{-j\beta r} \vec{a}_r \\ &+ \frac{j\beta \tilde{I} l}{4\pi r \omega\epsilon} \beta \left[1 + \frac{1}{j\beta r} - \frac{1}{\beta^2 r^2}\right] \sin\theta e^{-j\beta r} \vec{a}_\theta \end{aligned}$$

Substituting $\frac{\beta}{\omega\epsilon} = \eta$, we obtain the desired equation.

Exercise 11.6

$$\tilde{H} = j \frac{\beta \tilde{I} l}{4\pi r} \sin\theta e^{-j\beta r} \vec{a}_\theta, \quad \nabla \cdot \vec{B} = 0, \quad \nabla \cdot \tilde{H} = 0 \text{ satisfied}$$

$$\tilde{E} = j \frac{\beta \tilde{I} l}{4\pi r} \eta \sin\theta e^{-j\beta r} \vec{a}_\theta. \quad \text{For } \nabla \cdot \tilde{D} = 0, \quad \nabla \cdot \tilde{E} \text{ must be zero.}$$

However, $\nabla \cdot \tilde{E} = \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} (\sin\theta \tilde{E}_\theta)$ is not zero.

Exercise 11.7

$$\vec{a}_\phi \cdot \vec{a}_r = \sin\theta$$

$$(\vec{a}_\phi \cdot \vec{a}_r) \vec{a}_\phi = \sin\theta \vec{a}_\phi$$

$$a \vec{a}_\phi + \vec{R} = r \vec{a}_r \Rightarrow \vec{R} = r \vec{a}_r - a \vec{a}_\phi$$

$$R^2 = r^2 + a^2 - 2ar \vec{a}_r \cdot \vec{a}_\phi$$

$$= r^2 + a^2 - 2ar \sin\theta \cos(\phi - \phi')$$

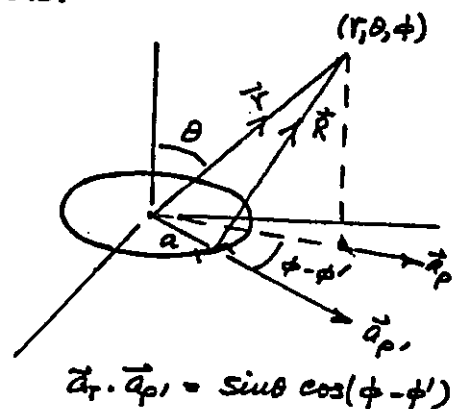
For $r \gg a$, $R^2 \approx r^2 - 2ar \sin\theta \cos(\phi - \phi')$

$$\text{or } R = r \left[1 - \frac{2ar}{r^2} \sin\theta \cos(\phi - \phi')\right]^{1/2} = r - a \sin\theta \cos(\phi - \phi') \quad [11.53]$$

$$\int_0^{2\pi} [1 + j\beta a \sin\theta \cos(\phi - \phi')] \cos(\phi - \phi') d\phi'$$

$$= \int_0^{2\pi} \cancel{\cos(\phi - \phi')} d\phi' + j\beta a \sin\theta \int_0^{2\pi} \cancel{\cos^2(\phi - \phi')} d\phi'$$

$$= j\beta a \pi \sin\theta \quad (11.57)$$



Exercise 11.8

$$\omega = 3 \times 10^6 \text{ rad/s} \quad \tilde{I} = 100 e^{-j\pi/6} \text{ A}, \quad M = \pi \tilde{I}^2 \tilde{r} = 3.142 e^{j\pi/6}$$

$$\beta_0 = \frac{\omega}{c} = 0.01, \quad \eta_0 = 120\pi \quad \lambda_0 = \frac{2\pi}{\beta_0} = 628.32 \text{ m}$$

$$\frac{\omega \mu_0 \beta_0}{4\pi} \tilde{M} = 9.426 \times 10^{-3} e^{-j\pi/6}$$

$$R_{\text{rad}} = \frac{\pi}{6} \times 120\pi (0.01 \times 0.01)^4 \approx 197 \text{ } \mu\Omega$$

$$\tilde{E}_\phi = \frac{9.426}{r} \sin\theta e^{j0.01r} e^{-j\pi/6} \text{ mV/m}$$

$$\tilde{H}_\phi = -\frac{25}{r} \sin\theta e^{j0.01r} e^{-j\pi/6} \text{ } \mu\text{A/m}$$

$$\text{When } \omega = 30 \times 10^6 \text{ rad/s} \quad \beta = 0.1 \text{ rad/m} \quad \lambda_0 = 62.83 \text{ m} \quad \text{and } R_{\text{rad}} = 1.97 \text{ } \mu\Omega$$

Exercise 11.9

$$\begin{aligned} \int_{-l/2}^{l/2} (1 - az/l)(1 + j\beta z \cos\theta) dz &= \int_0^{l/2} (1 - \frac{z^2}{2} + j\beta z \cos\theta - j\frac{\beta^2}{2} z^2 \cos\theta) dz \\ &= a \left[z - \frac{z^3}{3} + j\beta \frac{z^2}{2} \cos\theta - j\frac{\beta^2}{2} \frac{z^3}{3} \cos\theta \right]_0^{l/2} \\ &= \frac{l}{2} + j \frac{\beta l}{2} \cos\theta \end{aligned}$$

$$\text{Thus, } \tilde{A}_z = \frac{\mu}{8\pi r} \tilde{I} l (1 + j \frac{\beta l}{2} \cos\theta) e^{-j\beta r} \quad (11.70)$$

$$\tilde{H}_\phi = \frac{j\beta}{\mu} \sin\theta \tilde{A}_z = j \frac{\beta \tilde{I} l}{8\pi r} (1 + j \frac{\beta l}{2} \cos\theta) \sin\theta e^{-j\beta r} \quad (11.71)$$

$$\text{and } \tilde{E}_\theta = \eta \tilde{H}_\phi$$

Exercise 11.10

$$P_{\text{rad}}|_{\text{mono}} = \frac{1}{2} P_{\text{rad}}|_{\text{dipole}} \Rightarrow$$

$$\begin{aligned} \int_0^{\pi/2} \sin^3\theta d\theta &= \left[\frac{1}{3} \cos^3\theta - \cos\theta \right]_0^{\pi/2} \\ &= \frac{2}{3} \end{aligned}$$

$$\langle \hat{S} \rangle_{\text{mono}} = \frac{I^2 l^2 \beta^2 \eta}{128 \pi^2 r^2} \sin^2\theta \hat{a}_r$$

Finally,

$$R_{\text{rad}} = \frac{l^2 \beta^2 \eta}{48\pi} = \frac{\pi}{12} \eta \left(\frac{l}{\lambda} \right)^2 \Omega$$

$$\begin{aligned} P_{\text{rad}} &= \int_0^\pi \frac{I^2 l^2 \beta^2 \eta}{\pi^2 \cdot 128} \sin^3\theta d\theta \int_0^{2\pi} d\phi \\ &= \frac{I^2 l^2 \beta^2 \eta}{96\pi} \end{aligned}$$

Exercise 11.11

$$G = \frac{4\pi r^2 \langle \hat{S} \rangle}{P_{rad}} = \frac{4\pi r^2 \eta I_0^2}{8\pi^2 r^2} \frac{\cos^2(\frac{\pi}{2} \cos \theta)}{\sin^2 \theta} \cdot \frac{4\pi}{1.219 \eta I_0^2}$$

$$= 1.64 \frac{\cos^2(\frac{\pi}{2} \cos \theta)}{\sin^2 \theta}$$

when $\theta \rightarrow 90^\circ$, $G \rightarrow D \neq D = 1.64$ or 2.15 dB

Exercise 11.10 $f = 30 \text{ MHz}$, $\omega = 2\pi f = 1.885 \times 10^8 \text{ rad/s}$ $\beta = \frac{\omega}{c} = 0.628 \text{ rad/m}$

$$\lambda = \frac{2\pi}{\beta} = 10 \text{ m}, \quad \frac{\lambda}{2} = 5 \text{ m} \quad \theta = \frac{\pi}{6} \quad r = 3000 \text{ m}$$

From (11.77) $I_0 \approx 2 \text{ A}$ $\vec{E} = \frac{j120}{r} e^{-j\beta r} \frac{\cos(\frac{\pi}{2} \cos \theta)}{\sin \theta} \vec{a}_\theta$

$$\vec{H}_\phi = \frac{\vec{E}_\theta}{\eta} = \frac{j}{\pi r} e^{-j\beta r} \cos(\frac{\pi}{2} \cos \theta) / \sin \theta$$

$$R_{rad} = 73.14 \Omega \quad P_{rad} = \frac{1}{2} I_0^2 R_{rad} = 145.5 \text{ W}$$

Exercise 11.13

Math CAD:

$$\phi = 0, \frac{\pi}{500} \dots 2\pi$$

$$k=1 \quad \theta = \frac{\pi}{6} \quad \beta d = \frac{\pi}{5}$$

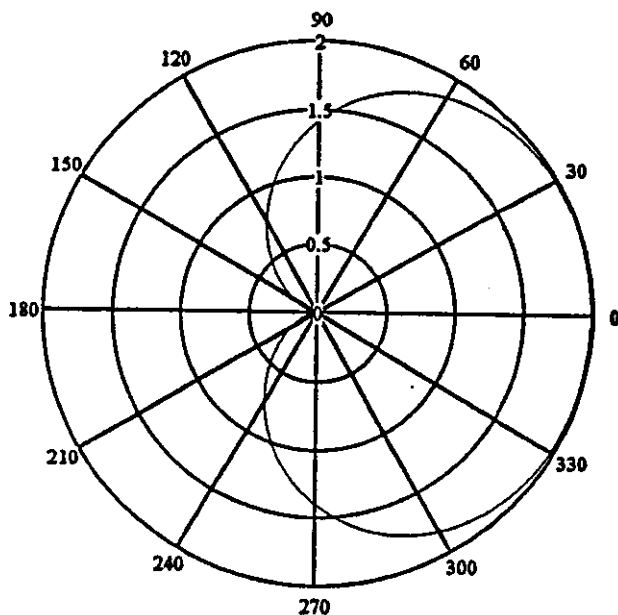
$$\delta = -\frac{\pi}{5}$$

$$\alpha(\phi) = \beta d \cdot \sin(\theta) \cdot \cos(\phi) + \delta$$

$$F(\theta) = 1$$

$$G(\phi) = \sqrt{(1 + k \cdot \cos(\alpha(\phi)))^2 + (k \cdot \sin(\alpha(\phi)))^2}$$

Plot is $F_\theta \cdot G\phi$ as a function of ϕ



Exercise 11.14

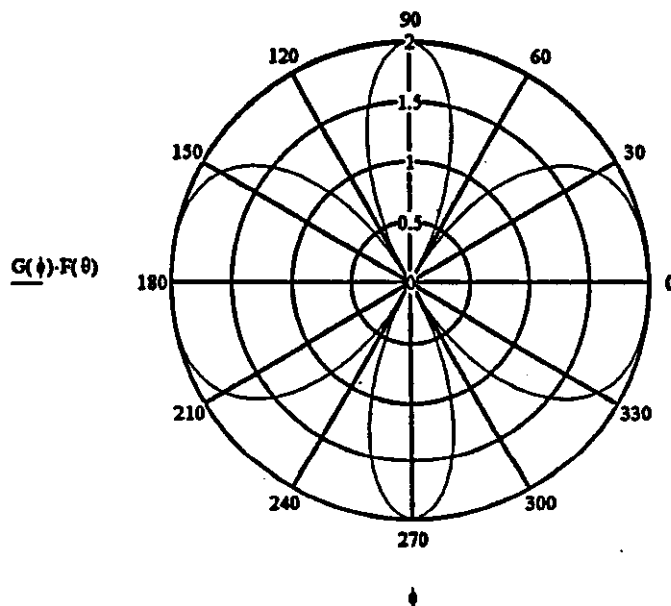
$$\phi := 0, \frac{\pi}{500} \dots 2\pi$$

$$k := 1 \quad \theta := \frac{\pi}{2} \quad \beta d := 2\pi \quad \delta := 0$$

$$u(\phi) := \beta d \cdot \sin(\theta) \cdot \cos(\phi) + \delta$$

$$\text{Note that } F(\theta) := 1$$

$$G(\phi) := \sqrt{(1 + k \cos(u(\phi)))^2 + (k \sin(u(\phi)))^2}$$



Exercise 11.15

$$\phi := 0, \frac{\pi}{500} \dots 2\pi$$

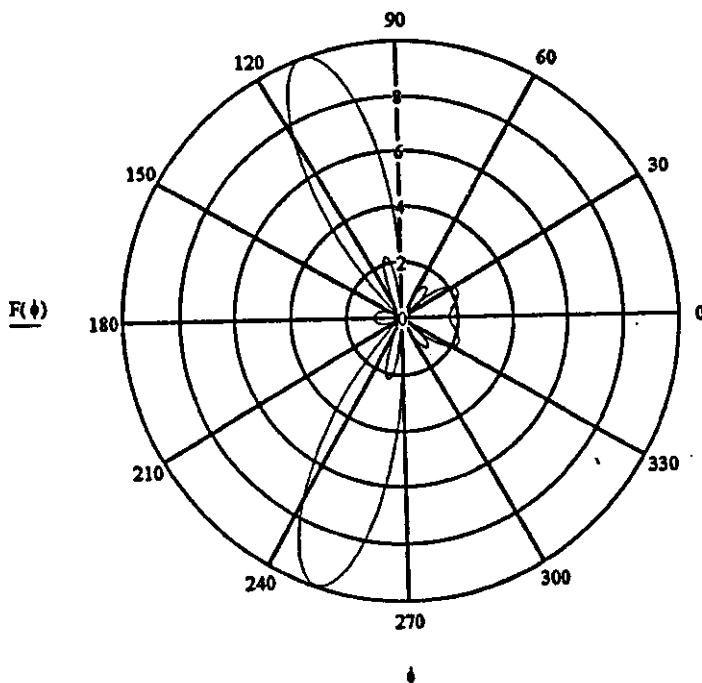
$$\beta d := \frac{\pi}{2}$$

$$\delta := \frac{\pi}{6}$$

$$n := 10$$

$$u(\phi) := \beta d \cdot \cos(\phi) + \delta$$

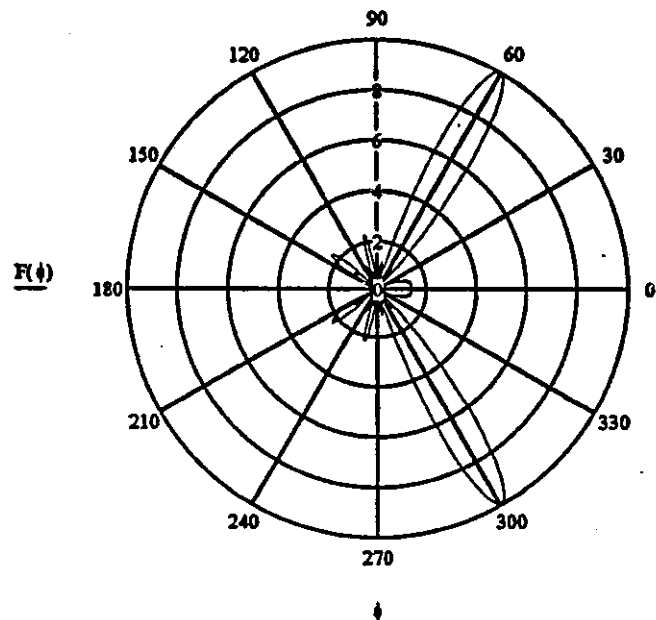
$$F(\phi) := \frac{\sin(n \cdot u(\phi) \cdot 0.5)}{\sin(u(\phi) \cdot 0.5)}$$



Exercise 11.16

$$\phi := 0, \frac{\pi}{500} \dots 2\pi \quad \beta d := \pi \quad \delta := -\left(\frac{\pi}{2}\right) \quad n := 10 \quad \alpha(\phi) := \beta d \cdot \cos(\phi) + \delta$$

$$F(\phi) := \frac{\sin(n \cdot \alpha(\phi) - 0.5)}{\sin(\alpha(\phi) - 0.5)}$$



Exercise 11.17

$$f = 300 \text{ MHz} \quad \omega = 2\pi f = 1.885 \times 10^9 \text{ rad/s}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} \quad \sigma_{cu} = 5.8 \times 10^7 \text{ S/m} \quad a = 406.5 \text{ } \mu\text{m}$$

$$\alpha = \sqrt{\pi f \mu_0 \sigma_{cu}} = 262.092 \times 10^3 \text{ NP/m} \Rightarrow \delta = \frac{1}{\alpha} = 3.815 \text{ } \mu\text{m}$$

$$\text{Since } a \gg \delta, \quad A_{cu} = 2\pi a \delta \quad \ell = \frac{\lambda}{10} \quad \lambda = \frac{2\pi}{\beta} = \frac{c}{f} = 1$$

$$\text{Hence } R_c = \frac{\lambda/10}{2\pi a \delta \sigma_{cu}} = 0.177 \Omega$$

$$\text{Short antenna: } R_{rad} = \frac{2\pi}{12} \eta \left(\frac{\ell}{\lambda}\right)^2 = 1.974 \Omega$$

$$\eta = \frac{R_{rad}}{R_{rad} + R_c} = 0.9177 \text{ or } 91.77\%$$

Exercise 11.18 $f = 600 \text{ MHz}$ $\omega = 2\pi f$ $\lambda = \frac{c}{f} = 0.5 \text{ m}$ $l = \frac{\lambda}{2} = 0.25 \text{ m}$

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{1}{\eta f \mu_0 \sigma_{\text{Cu}}}} = 2.698 \mu\text{m}, \quad a = 406.5 \mu\text{m} \quad a \gg \delta \quad A_{\text{Cu}} = 2\pi a \delta$$

$$R_c = \frac{l}{2\pi a \delta \sigma_{\text{Cu}}} = 0.626 \Omega \quad R_{\text{rad}} = 73.14 \Omega \quad \eta = \frac{73.14}{73.14 + 0.626} = 0.9915$$

or 99.15%

Exercise 11.19

$$A_{et} = \frac{\lambda^2}{4\pi} G_t = \frac{3^2}{4\pi} \times 1.64 = 1.175 \quad , \quad P_R = 10 \times 10^{-3} \times (1.175)^2 \left(\frac{1}{3 \times 25000} \right)^2$$

$$A_{er} = \frac{\lambda^2}{4\pi} G_R = \frac{3^2}{4\pi} \times 1.64 = 1.175 \quad = 2.45 \mu\text{W}$$

Exercise 11.20 $D_T = 10^{1.2} = 15.85$ $D_L = 10^2 = 100$

Since $\theta = 90^\circ$ $G_R = D_T = 15.85$ $G_L = D_L = 100$ $R = 100\lambda$ $P_T = 10 \mu\text{W}$

From (11.112), $P_{\text{rad}} = \frac{P_R}{G_t G_R (\lambda/4\pi R)^2} = 9.96 \text{ mW}$

Exercise 11.21 $P_R = \frac{1}{4\pi} \left(\frac{G\lambda}{4\pi R^2} \right)^2 A_{eo} P_{\text{rad}}$

$$A_e = \frac{\lambda^2}{4\pi} G \Rightarrow G = \frac{4\pi A_e}{\lambda^2} \text{ . Hence, } P_R = \frac{1}{4\pi} \left(\frac{A_e}{\lambda R^2} \right)^2 A_{eo} P_{\text{rad}}$$

Exercise 11.22 From Example 11.8,

$$P_R = \frac{1}{4\pi} \left[\frac{100 \times 0.1}{4\pi \times 2000^2} \right]^2 4 \times 100 \times 10^3 = 1.26 \text{ nW}$$

Problem 11.1.

$$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t} \Rightarrow \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_\phi) - \frac{\partial E_\theta}{\partial \phi} \right] = -\mu \frac{\partial H_r}{\partial t}$$

$$\frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \phi} E_r \right] - \frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) = -\mu \frac{\partial H_\phi}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_\theta) - \frac{1}{r} \frac{\partial}{\partial \theta} E_r = -\mu \frac{\partial H_\phi}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \Rightarrow \nabla \cdot \vec{H} = 0 \Rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 H_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta H_\theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} H_\phi = 0$$

$$\nabla \times \vec{H} = \epsilon \frac{\partial \vec{E}}{\partial t} \Rightarrow \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta H_\phi) - \frac{\partial H_\theta}{\partial \phi} \right] = \epsilon \frac{\partial E_r}{\partial t}$$

$$\frac{1}{r \sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{1}{r} \frac{\partial}{\partial r} (r H_\theta) = \epsilon \frac{\partial E_\theta}{\partial t}$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r H_\theta) - \frac{1}{r} \frac{\partial}{\partial \theta} H_r = \epsilon \frac{\partial E_\phi}{\partial t}$$

$$\nabla \cdot \vec{D} = 0 \Rightarrow \nabla \cdot \vec{E} = 0 \Rightarrow \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r) + \frac{1}{r \sin \theta} \left[\frac{\partial}{\partial \theta} (\sin \theta E_\theta) + \frac{\partial E_\phi}{\partial \phi} \right] = 0$$

Problem 11.2

$$\tilde{V} = \frac{q}{4\pi\epsilon} \left[\frac{e^{-j\beta r_1}}{r_1} - \frac{e^{-j\beta r_2}}{r_2} \right]$$

$$r_1 \approx r - \frac{l}{2} \cos \theta, \quad r_2 = r + \frac{l}{2} \cos \theta$$

$$\tilde{V} = \frac{q}{4\pi\epsilon} e^{-j\beta r} \left[\frac{e^{j\beta \frac{l}{2} \cos \theta}}{r - \frac{l}{2} \cos \theta} - \frac{e^{-j\beta \frac{l}{2} \cos \theta}}{r + \frac{l}{2} \cos \theta} \right]$$

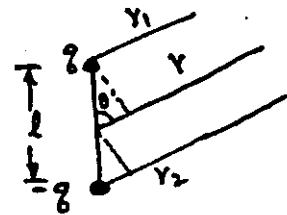
$$\text{Thus, } \tilde{V} = \frac{q}{4\pi\epsilon r^2} e^{-j\beta r} \left[(r + \frac{l}{2} \cos \theta) e^{j\beta \frac{l}{2} \cos \theta} - (r - \frac{l}{2} \cos \theta) e^{-j\beta \frac{l}{2} \cos \theta} \right]$$

$$= \frac{q}{4\pi\epsilon r^2} e^{-j\beta r} \left[j\beta r \sin(\beta \frac{l}{2} \cos \theta) + l \cos \theta \cos(\beta \frac{l}{2} \cos \theta) \right]$$

$$\text{when } \beta l \ll r, \quad \tilde{V} = \frac{q}{4\pi\epsilon r^2} e^{-j\beta r} \left[j\beta l r \cos \theta + l \cos \theta \right]$$

[$\sin \theta \approx \theta$]

$$\text{Finally, } \tilde{V} = \frac{ql}{4\pi\epsilon} \left[\frac{1}{r^2} + j \frac{\beta}{r} \right] \cos \theta e^{-j\beta r}$$



$$r_1 r_2 \approx r^2 - \left(\frac{l}{2} \cos \theta \right)^2 \approx r^2$$

Problem 11.3

$$\tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{A}} - \nabla\tilde{V} \quad \text{But } \nabla\cdot\tilde{\mathbf{A}} = -j\omega\mu\epsilon\tilde{V}$$

$$\text{Hence } \tilde{\mathbf{E}} = -j\omega\tilde{\mathbf{A}} + \frac{\nabla(\nabla\cdot\tilde{\mathbf{A}})}{j\omega\mu\epsilon} = -j\omega\tilde{\mathbf{A}} - j\frac{\omega}{\beta^2}\nabla(\nabla\cdot\tilde{\mathbf{A}}) \quad \beta = \omega\sqrt{\mu\epsilon}$$

$$\text{Thus: } \tilde{\mathbf{E}} = -j\omega\left[\tilde{\mathbf{A}} + \frac{1}{\beta^2}\nabla(\nabla\cdot\tilde{\mathbf{A}})\right]$$

Problem 11.4 $\tilde{A} = \sin\beta y \tilde{a}_x$, The wave equation in a source-free

$$\text{medium is } \nabla^2\tilde{A} + \omega^2\mu\epsilon\tilde{A} = 0 \quad \nabla^2\tilde{A} = \frac{\partial^2\tilde{A}}{\partial x^2} + \frac{\partial^2\tilde{A}}{\partial y^2} + \frac{\partial^2\tilde{A}}{\partial z^2} = -\beta^2\tilde{A}_x$$

$$\text{Thus, } \beta^2 = \omega^2\mu\epsilon$$

$$\tilde{\mathbf{B}} = \nabla \times \tilde{\mathbf{A}} = -\beta \cos\beta y \tilde{a}_z \Rightarrow \tilde{H}_z = -\frac{\beta}{\mu} \cos\beta y$$

$$\nabla \times \tilde{\mathbf{H}} = j\omega\epsilon\tilde{\mathbf{E}} \Rightarrow \tilde{\mathbf{E}} = \frac{\beta^2}{j\omega\mu\epsilon} \sin\beta y = -j\omega \sin\beta y$$

Problem 11.5 From (11.36) and (11.41), $\frac{\tilde{H}_\phi}{\tilde{A}_z} = \frac{j\beta}{\mu} \sin\theta \Rightarrow \tilde{H}_\phi = \frac{j\beta}{\mu} \sin\theta \tilde{A}_z$ Problem 11.6

$$R_{\text{rad}} = 80\pi^2 (0.1)^2 = 7.9\Omega \quad P_{\text{rad}} = 500\text{W}$$

$$P_{\text{rad}} = \frac{1}{2} I^2 R_{\text{rad}} \Rightarrow I = \sqrt{\frac{2 \times 500}{7.9}} = 11.25\text{A}$$

Problem 11.7

$$\text{From (11.43), } \frac{\beta I}{4\pi} \eta = 6 \times 10^3 \times 10 \times 10^3 = 60 \quad r = 10\text{ km} \\ \eta = 180\pi$$

$$\tilde{E}_\theta = j \frac{60}{r} \sin\theta e^{-j\beta r}$$

$$\langle \hat{S}_r \rangle = \frac{1}{2} \text{Re}[\tilde{E}_\theta \tilde{H}_\phi^*] \cdot \tilde{a}_r$$

$$\tilde{H}_\phi = j \frac{1}{2\pi r} \sin\theta e^{-j\beta r}$$

$$= \frac{15}{\pi r^2} \sin^2\theta \quad \text{W/m}^2$$

$$P_T = \frac{15}{\pi} \int_0^\pi \sin^3\theta d\theta \int_0^{2\pi} d\phi = 40\text{W}$$

Problem 11.8

$$\omega = 300 \times 10^6 \text{ rad/s} \quad \beta = \omega/c = 1 \text{ rad/m} \quad \tilde{I} = 10\text{A (max)}$$

$$\tilde{E}_\theta = 15.5 e^{-j77.34^\circ} \text{ mV/m} \quad \text{at } r = 300\text{m}$$

$$\lambda = 2\pi \quad r = 300\text{m}$$

$$\tilde{H}_\phi = 41.11 e^{-j77.34^\circ} \text{ } \mu\text{A/m}$$

$$\langle \hat{S}_r \rangle = 0.32 \text{ } \mu\text{W/m}^2$$

Problem 11.9

$$\int_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}} \left(\frac{e^{j\beta(1+\cos\theta)z}}{e^{j\beta(1+\cos\theta)z}} + \frac{e^{-j\beta(1-\cos\theta)z}}{e^{-j\beta(1-\cos\theta)z}} \right) dz = \left[\frac{e^{j\beta z} e^{j\beta z \cos\theta}}{j\beta(1+\cos\theta)} - \frac{e^{-j\beta z} e^{j\beta z \cos\theta}}{j\beta(1-\cos\theta)} \right]_{-\frac{\lambda}{4}}^{\frac{\lambda}{4}}$$

$$= \frac{j e^{\frac{j\pi}{2} \cos\theta} + j e^{-\frac{j\pi}{2} \cos\theta}}{j\beta(1+\cos\theta)} - \frac{-j e^{\frac{j\pi}{2} \cos\theta} - j e^{-\frac{j\pi}{2} \cos\theta}}{j\beta(1-\cos\theta)} \quad \text{Let } \frac{\pi}{2} \cos\theta = \alpha$$

Then

$$= \frac{\frac{j\alpha}{e + e^{-j\alpha}} - \cos\theta \frac{j\alpha}{e - \cos\theta e} - \cos\theta \frac{-j\alpha}{e + e^{-j\alpha}} + \cos\theta \frac{j\alpha}{e + \cos\theta e} - j\alpha}{\beta(1 - \cos^2\theta)}$$

$$= \frac{4 \cos\alpha}{\beta \sin^2\theta} = \frac{4 \cos(\frac{\pi}{2} \cos\theta)}{\beta \sin^2\theta}$$

Problem 11.10 Sum up the fields due to many Hertzian dipoles stacked upon each other. Thus. $l = dz$

$$\tilde{H}_\phi = \frac{j\beta}{4\pi} \sin\theta \int_{-\lambda/4}^{\lambda/4} \frac{I_0}{\partial R} (e^{j\beta z} + e^{-j\beta z}) e^{j\beta R} dz, \text{ where } R = r - z \cos\theta$$

$$= \frac{j\beta I_0}{8\pi r} \sin\theta \int_{-\lambda/4}^{\lambda/4} (e^{j\beta z} + e^{-j\beta z}) e^{j\beta z \cos\theta} e^{-j\beta r} dz$$

Problem 11.9: $\int_{-\lambda/4}^{\lambda/4} (e^{j\beta z} + e^{-j\beta z}) e^{j\beta z \cos\theta} dz = \frac{4 \cos(\frac{\pi}{2} \cos\theta)}{\beta \sin^2\theta}$

Hence, $\tilde{H}_\phi = j \frac{I_0}{2\pi r} e^{-j\beta r} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta}$

Problem 11.11 \tilde{E}_θ is given in (11.77), \tilde{H}_ϕ in (11.76b) and $\langle \hat{S} \rangle$ in (11.78).

These equations are true for a $\lambda/4$ monopole above $z \geq 0$ plane.

However, power radiated will be $\frac{1}{2}$ of that the dipole.

Hence $P_{rad}|_{\text{mono}} = \frac{1.219}{8\pi} \eta I_0^2$ and $R_{rad} = \frac{1.219}{4\pi} \eta = 36.57 \Omega$ in free space

Problem 11.12 $f = 50 \text{ MHz}$ $\omega = 2\pi f = 3.142 \times 10^8 \text{ rad/s}$ $\beta = \frac{\omega}{c} = 1.047 \text{ m}^{-1}$

$\lambda = 2\pi/\beta = 6 \text{ m} \Rightarrow l = 3 \text{ m}$ $\eta = 120\pi \Omega$ $I_0 = 5 \text{ A}$

$$\vec{E}_\theta = \frac{j\eta}{2\pi r} I_0 e^{-j\beta r} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta} = \frac{j25\pi}{r} e^{-j1.047r} \frac{\cos(\frac{\pi}{2} \cos\theta)}{\sin\theta}$$

$$\vec{H}_\phi = j \frac{0.796}{r} e^{-j1.047r} \cos(\frac{\pi}{2} \cos\theta) / \sin\theta, \quad R_{\text{rad}} = 73.14 \Omega$$

$$P_{\text{rad}} = \frac{1}{2} \times 5^2 \times 73.14 = 914.25 \text{ W}$$

Problem 11.13 $\vec{J}_s = \frac{I_0}{2\pi b} \vec{a}_z \text{ A/m}$

$$dP = \frac{1}{2} I_0^2 \frac{dz}{2\pi b \sigma \delta} \Rightarrow P = \frac{1}{2} I_0^2 \int_{-\lambda/4}^{\lambda/4} \frac{dz}{2\pi b \sigma \delta} = \frac{1}{2} I_0^2 \left[\frac{\lambda}{4\pi b \sigma \delta} \right] = \frac{1}{2} I_0^2 R$$

where $R = \frac{\lambda/2}{2\pi b \sigma \delta}$

Problem 11.14 Continue from Problem 11.13

$$dP = \frac{1}{2} I_0^2 \cos^2 \beta z \frac{dz}{2\pi b \sigma \delta}$$

$$P = \frac{1}{2} I_0^2 \frac{1}{2\pi b \sigma \delta} \int_{-\lambda/4}^{\lambda/4} \cos^2 \beta z dz = \frac{1}{2} I_0^2 \left[\frac{\lambda/4}{2\pi b \sigma \delta} \right] \Rightarrow R = \frac{\lambda/4}{2\pi b \sigma \delta}$$

Problem 11.15 Let $\theta = 90^\circ$, from (11.77)

$$|E| = \frac{\eta I_0}{2\pi r} \Rightarrow \frac{120\pi I_0}{2\pi \times 100 \times 10^{-3}} = 25 \times 10^3 \Rightarrow I_0 = 41.667 \text{ A}$$

From (11.79), $P_{\text{rad}} = \frac{1.219}{4\pi} \times 120\pi \times 41.667^2 = 63.5 \text{ kW}$

Problem 11.16 $|E|$ still the same as in Problem 11.15.

$I_0 = 41.667 \text{ A}$ and $P_{\text{rad}} = \frac{1}{2} [P_{\text{rad}}]_{\text{Half-wave}} = 31.75 \text{ kW}$

Problem 11.17 $\frac{l}{\lambda} = 0.1$ $\beta l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{10} = \pi/5$

$P_{rad} = \frac{1}{2} I_0^2 R_{rad} = \frac{1}{2} I_0^2 \frac{2\pi}{\lambda} \times 120\pi \times \left(\frac{l}{10}\right)^2$ When $P_{rad} = 100 \text{ W}$, $I_0 = 10.67 \text{ A}$

$|E| = \frac{\beta I_0 l}{8\pi r} \eta = \frac{\pi}{5} \cdot \frac{10.67}{8\pi} \cdot \frac{120\pi}{10 \times 10^3} = 9.49 \text{ mV/m}$

Problem 11.18 $I_0 = \sqrt{100 \times \frac{4\pi}{1.219} \times \frac{1}{120\pi}} = 1.654 \text{ A}$

$|E| = \frac{\eta I_0}{2\pi r} = \frac{120\pi \times 1.654}{2\pi \times 10 \times 10^3} = 9.92 \text{ mV/m}$

Problem 11.19 Since $P_{rad}/\text{mono} = \frac{1}{2} P_{rad}/\text{Half-wave}$, $I_0 = \sqrt{2} \times 1.654 = 2.34 \text{ A}$

Thus, $|E| = \sqrt{2} \times 9.92 = 14.03 \text{ mV/m}$

Problem 11.20 $f = 100 \text{ MHz}$, $\omega = 2\pi f$, $\lambda = c/f = 3 \text{ m}$ $\beta\lambda = 2\pi \Rightarrow \beta = 2\pi/3$

From (11.63a): $100 = \frac{4\pi^2}{3} \times 120\pi \times \left(\frac{M}{\lambda}\right)^2 \Rightarrow M = 0.28 \text{ A}\cdot\text{m}^2$

$|E| = \frac{\omega \mu_0 \beta}{4\pi r} M = 368 \text{ mV/m}$. $r = 10 \text{ km}$

Problem 11.21

$$\tilde{A}_2 = \frac{\mu I_0}{2\pi r} e^{-j\beta r} \left[\int_0^{l/2} \sin\left(\frac{\beta l}{2} - \beta z\right) e^{j\beta z \cos\theta} dz + \int_{-l/2}^0 \sin\left(\frac{\beta l}{2} + \beta z\right) e^{j\beta z \cos\theta} dz \right]$$

$$= \frac{\mu I_0}{2\pi r} e^{-j\beta r} \frac{\cos(\frac{\beta l}{2} \cos\theta) - \cos(\beta l/2)}{\beta \sin^2\theta}$$

$\tilde{H}_\phi = \frac{j\beta}{\mu} \sin\theta \tilde{A}_2 = \frac{j I_0 e^{-j\beta r}}{2\pi r \sin\theta} [\cos(\frac{\beta l}{2} \cos\theta) - \cos(\frac{\beta l}{2})]$

$\tilde{E}_\theta = \eta \tilde{H}_\phi = \frac{j I_0 \eta}{2\pi r} \frac{e^{-j\beta r}}{\sin\theta} [\cos(\frac{\beta l}{2} \cos\theta) - \cos(\frac{\beta l}{2})]$

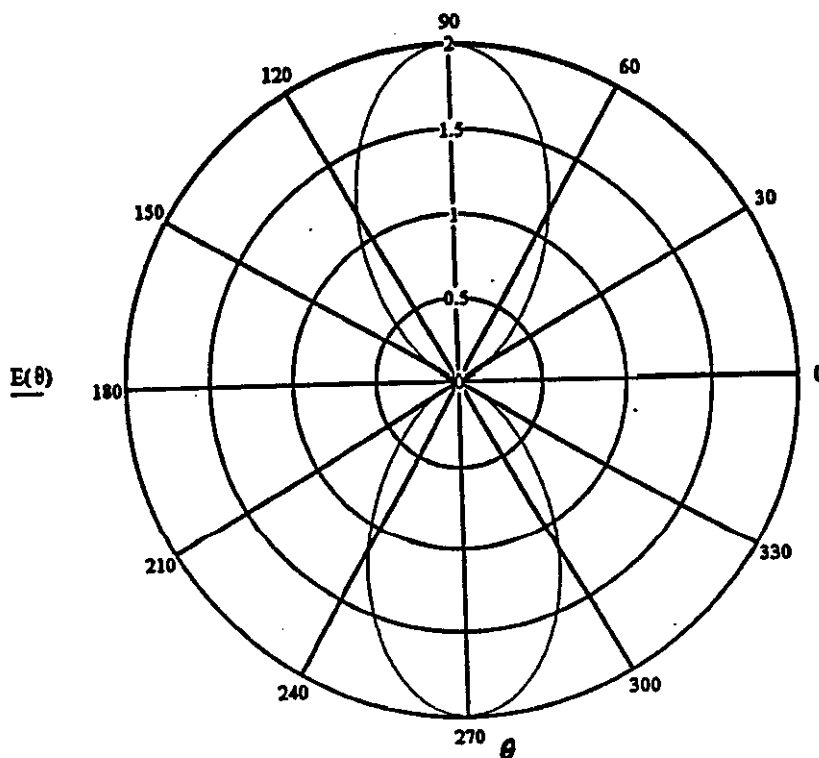
$\langle \hat{S}_r \rangle = \frac{1}{2\eta} E_\theta^2 = \frac{I_0^2 \eta}{8\pi^2 r^2} \left[\frac{\cos(\frac{\beta l}{2} \cos\theta) - \cos(\beta l/2)}{\sin\theta} \right]^2$

Problem 11.22

$$\theta := 0, \frac{\pi}{500} .. 2 \cdot \pi$$

$$E(\theta) := \left| \frac{\cos(\pi \cos(\theta)) + 1}{\sin(\theta)} \right|$$

Field plot of a
Full-wave Antenna

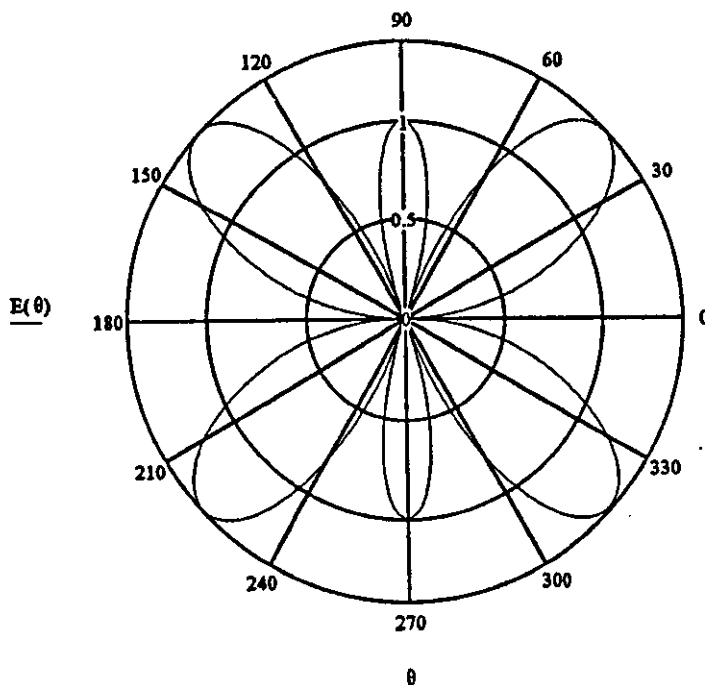


Problem 11.23

$$\theta := 0, \frac{\pi}{500} .. 2 \cdot \pi$$

$$E(\theta) := \left| \frac{\cos(\pi \cdot 1.5 \cdot \cos(\theta))}{\sin(\theta + 0.0001)} \right|$$

Field plot of a one-and-a-half
wavelength antenna

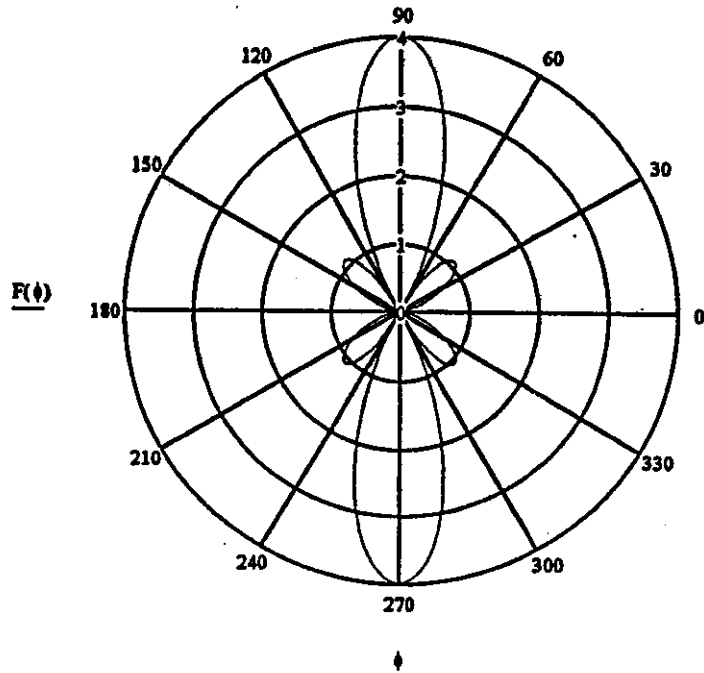


$$\phi := 0, \frac{\pi}{500} \dots 2\pi \quad \beta d := \pi \quad \delta := 0 \quad n := 4 \quad \alpha(\phi) := \beta d \cos(\phi) + \delta$$

$$F(\phi) := \left| \frac{\sin(n\alpha(\phi) - 0.5)}{\sin(\alpha(\phi) - 0.5)} \right|$$

Broadside array pattern of a 4-element half-wave dipole array in the xy-plane when currents are in-phase and spacing is half-wavelength.

Problem 11.24

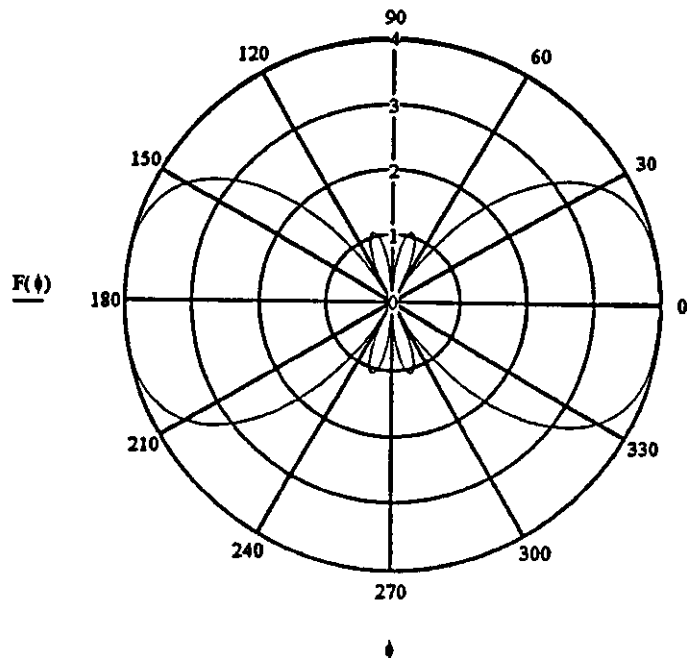


$$\phi := 0, \frac{\pi}{500} \dots 2\pi \quad \beta d := \pi \quad \delta := -\pi \quad n := 4 \quad \alpha(\phi) := \beta d \cos(\phi) + \delta$$

$$F(\phi) := \left| \frac{\sin(n\alpha(\phi) - 0.5)}{\sin(\alpha(\phi) - 0.5)} \right|$$

Endfire array pattern of a 4-element half-wave dipole array in the xy-plane when currents are -180° out of phase and spacing is half-wavelength.

Problem 11.25



$$\phi := 0, \frac{\pi}{500} \dots 2\pi$$

$$\beta d := \frac{\pi}{2}$$

$$\delta := -0.6\pi$$

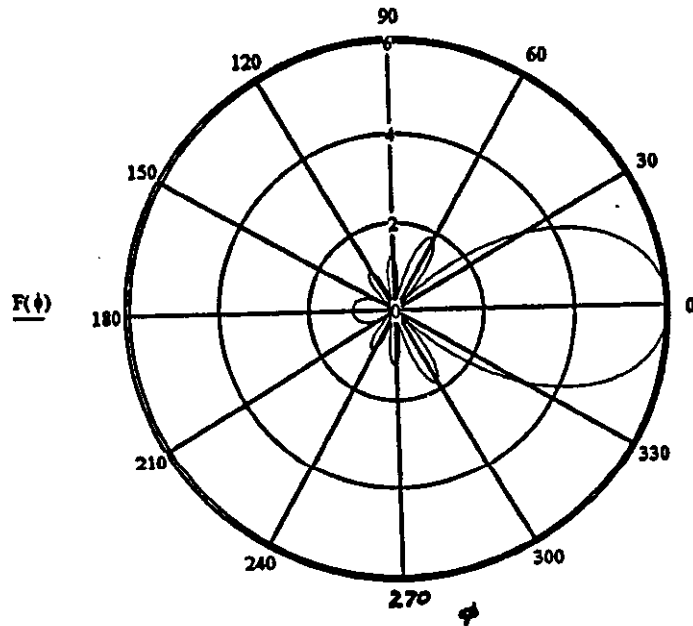
$$n := 8$$

$$\alpha(\phi) := \beta d \cos(\phi) + \delta$$

$$F(\phi) := \left| \frac{\sin(n\alpha(\phi) \cdot 0.5)}{\sin(\alpha(\phi) \cdot 0.5)} \right|$$

Endfire array pattern of an 8-element half-wave dipole array in the xy-plane when currents are -108° out of phase and spacing is quarter-wavelength.

Problem 11.26



$$\phi := 0, \frac{\pi}{500} \dots 2\pi$$

$$\beta d := \frac{\pi}{2}$$

$$\delta := -\frac{\pi}{2}$$

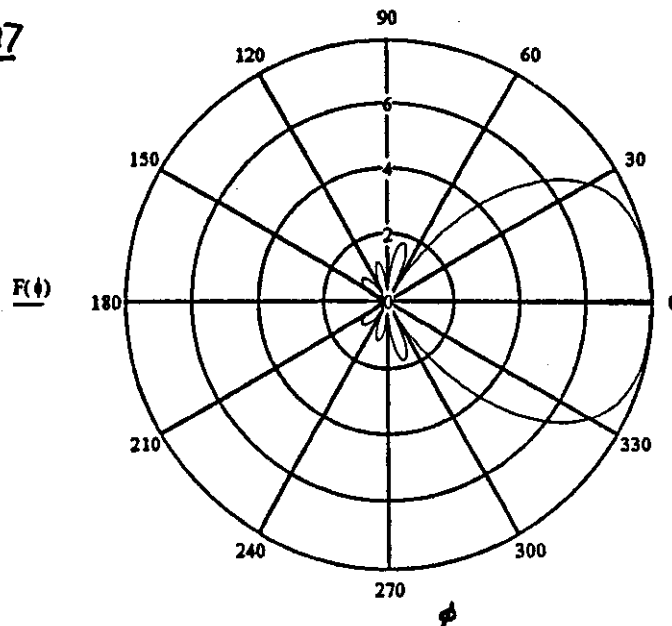
$$n := 8$$

$$\alpha(\phi) := \beta d \cos(\phi) + \delta$$

$$F(\phi) := \left| \frac{\sin(n\alpha(\phi) \cdot 0.5)}{\sin(\alpha(\phi) \cdot 0.5)} \right|$$

Endfire array pattern of an 8-element half-wave dipole array in the xy-plane when currents are -90° out of phase and spacing is quarter-wavelength.

Problem 11.27



Problem 11.28 $\tilde{E}_\theta = \frac{15}{r} I_0 \text{ V/m}$, $\tilde{H}_\phi = \frac{15}{\eta_0 r} I_0 \text{ A/m}$, $\langle \hat{S}_r \rangle = \frac{225}{\eta_0 r^2} I_0^2 \text{ W/m}^2$

$$P_{\text{rad}} = \frac{225}{\eta_0} I_0^2 \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi = 7.5 I_0^2, \text{ For } P_{\text{rad}} = 75 \text{ kW}, I_0 = 100 \text{ A}$$

$$\eta_0 = 120\pi \Omega \quad \text{Omnidirectional antenna.}$$

Problem 11.29

$$\tilde{E}_\theta = \frac{15}{r} I_0 \sin\theta \text{ V/m}, \quad \tilde{H}_\phi = \frac{15}{\eta_0 r} I_0 \sin\theta \text{ A/m}, \quad \langle \hat{S}_r \rangle = \frac{225}{\eta_0 r^2} I_0^2 \sin^2\theta \text{ W/m}^2$$

$$P_{\text{rad}} = \frac{225}{\eta_0} I_0^2 \int_0^\pi \sin^3\theta d\theta \int_0^{2\pi} d\phi = 5 I_0^2 \quad \int_0^\pi \sin^3\theta d\theta = \frac{4}{3}$$

$$\text{For } P_{\text{rad}} = 75 \text{ kW}, I_0 = 122.47 \text{ A}$$

Problem 11.30 $f = 3 \text{ MHz}$ $\lambda = \frac{c}{f} = 100 \text{ m}$ $\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{100} \text{ rad/m}$

$$\sigma = 5.8 \times 10^7 \text{ S/m} \quad \delta = \sqrt{\frac{1}{\pi f \mu_0 \sigma}} = 39.15 \text{ } \mu\text{m}$$

$$b = 5 \text{ mm}$$

$$\text{Since } b \gg \delta, R_c = \frac{2\pi \times 0.5 \times 10^6}{5.8 \times 10^7 \times 2\pi \times 5 \times 10^{-3} \times 39.15} = 45.2 \text{ m}\Omega$$

$$\text{From (11.65b)}, R_{\text{rad}} = \frac{\pi}{6} \times 120\pi \times \left(\frac{2\pi}{100} \times 0.5 \right)^4 = 192.28 \text{ } \mu\Omega$$

$$\eta_0 \eta_c = \frac{R_{\text{rad}} \times 100}{R_{\text{rad}} + R_c} = 0.42 \%$$

Problem 11.31 From Prob. (11.31) $\langle \hat{S}_r \rangle = \frac{\eta I_0^2}{8\pi^2 r^2} \left[\frac{\cos(\beta l/2 \cos\theta) - \cos(\beta l/2)}{\sin\theta} \right]^2$

$$P_{\text{rad}} = \frac{I_0^2 \eta}{8\pi^2} \int_0^\pi \frac{[\cos(\beta l/2 \cos\theta) - \cos(\beta l/2)]^2}{\sin^3\theta} d\theta \int_0^{2\pi} d\phi$$

$$\text{or } R_{\text{rad}} = \frac{\eta}{2\pi} \int_0^\pi \frac{1}{\sin\theta} [\cos(\beta l/2 \cos\theta) - \cos(\beta l/2)]^2 d\theta$$

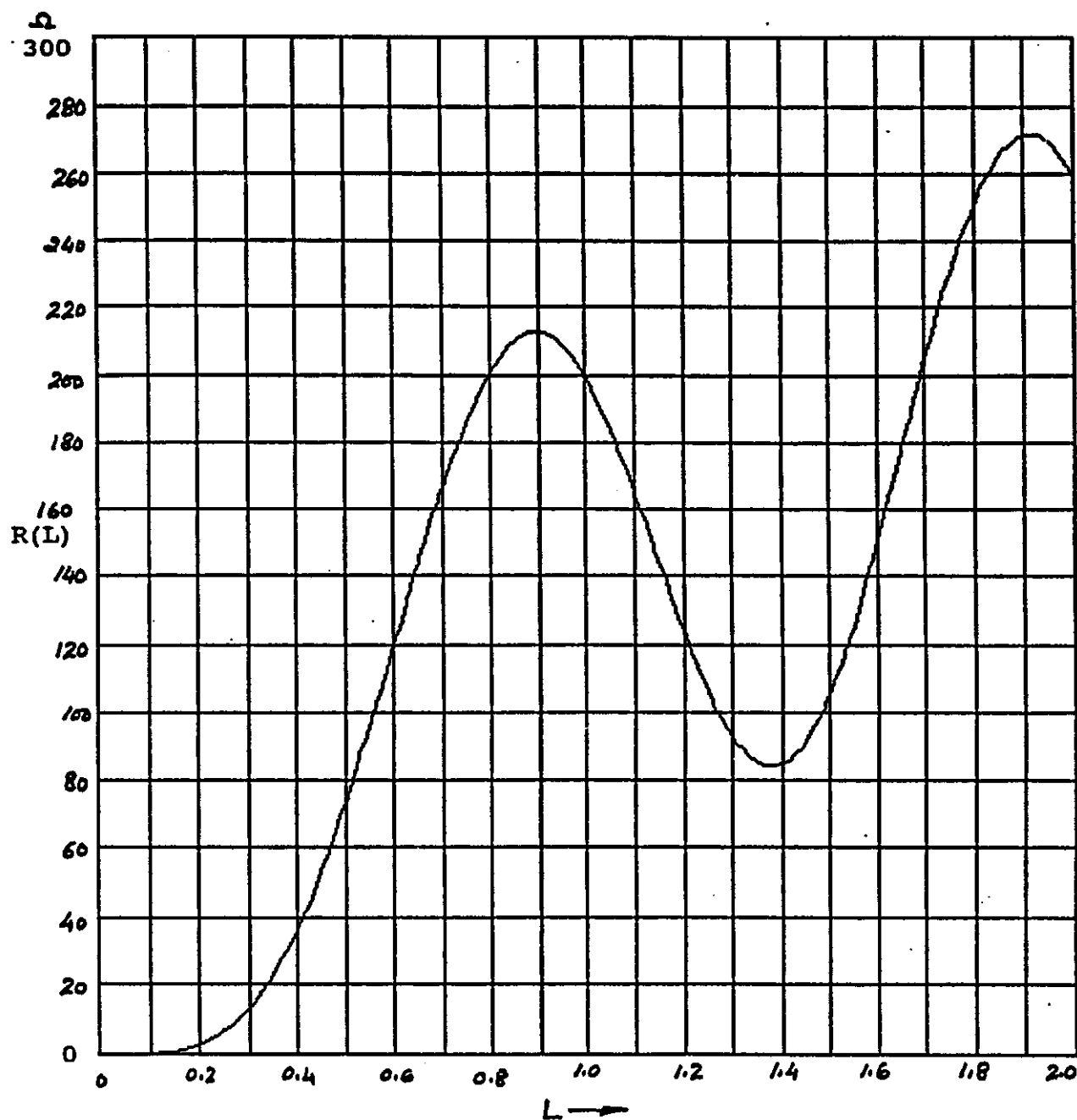
$$\text{When } l = \lambda \quad R_{\text{rad}} = \frac{3.318}{2\pi} \eta \Rightarrow R_{\text{rad}} \approx 200\Omega \text{ in free space}$$

$$\text{When } l = 2\lambda \quad R_{\text{rad}} = \frac{4.327}{2\pi} \eta \Rightarrow R_{\text{rad}} = 259.62\Omega \quad "$$

$$\text{When } l = 1.5\lambda \quad R_{\text{rad}} = \frac{1.758}{2\pi} \eta \Rightarrow R_{\text{rad}} \approx 105.5\Omega \quad "$$

$L := .1, .11 \dots 2$ where L is the length in wavelength $\eta := .120 \cdot \pi$

$$R(L) := 60 \cdot \left[\int_{0.001}^{\pi - 0.001} \left[\frac{(\cos(\pi \cdot L \cdot \cos(\theta)) - \cos(\pi \cdot L))^2}{\sin(\theta)} \right] d\theta \right]$$



Problem 11.33 $G_t = G_R = 10^2 = 100$ $f = 100 \text{ MHz} \Rightarrow \lambda = \frac{c}{f} = 3 \text{ m}$

$$R = \frac{300}{9} = 100 \lambda \quad P_{\text{rad}} = \frac{P_f}{G_t G_R \left(\frac{\lambda}{4\pi R}\right)^2} \quad \left(\frac{\lambda}{4\pi R}\right)^2 = \left(\frac{1}{400\pi}\right)^2$$

$$= \frac{10 \times 10^{-3}}{100 \times 100} \times (400\pi)^2 = 1.58 \text{ W}$$

Problem 11.34 $G_t = 1.64$ $G_R = 1.64$

$$P_{\text{rad}} = \frac{10 \times 10^{-3} \times (400\pi)^2}{1.64 \times 1.64} = 5.87 \text{ kW}$$

Problem 11.35 Since $P_r \propto \frac{1}{r^2}$, thus

$$(10 \times 10^{-3}) (500)^2 = (10 \times 10^{-6}) r_{\text{max}}^2 \Rightarrow r_{\text{max}} = 15.811 \text{ km}$$

Problem 11.36 $P_{\text{rad}} = 10 \text{ kW}$ $P_f = 3 \times 10^{-12} \text{ W}$

$$\lambda = \frac{3 \times 10^8}{5 \times 10^9} = 0.06 \text{ m} \quad G = 10^3 = 1000$$

Hence

$$r = \left[\frac{0.06^2 \times 1000^2 \times 1.5 \times 10 \times 10^3}{4\pi^3 \times 3 \times 10^{-12}} \right]^{1/4} \approx 9.8 \text{ km}$$